Paper Reference(s)

9801/01

Edexcel

Mathematics

Advanced Extension Award

Friday 29 June 2007 – Afternoon
Time: 3 hours

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil
Graph paper (ASG2)	
Answer Book (AB16)	

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, initials and signature.

Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

M26188A

1. (*a*) Write down the binomial expansion of $\dfrac{1}{(1-y)^2}$, $|y| < 1$, in ascending powers of y up to and including the term in y^3.

(1)

(*b*) Hence, or otherwise, show that

$$\frac{1}{4}\ \text{cosec}^4\left(\frac{\theta}{2}\right) = 1 + 2\cos\theta + 3\cos^2\theta + 4\cos^3\theta + \ldots + (r+1)\cos^r\theta + \ldots$$

and state the values of θ for which this result is not valid.

(4)

Find

(*c*)
$$1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots + \frac{(r+1)}{2^r} + \ldots,$$

(2)

(*d*)
$$1 - \frac{2}{2} + \frac{3}{2^2} - \frac{4}{2^3} + \ldots + (-1)^r\frac{(r+1)}{2^r} + \ldots .$$

(2)

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2. (*a*) On the same diagram, sketch $y = x$ and $y = \sqrt{x}$, for $x \geq 0$, and mark clearly the coordinates of the points of intersection of the two graphs.

(2)

(*b*) With reference to your sketch, explain why there exists a value a of x ($a > 1$) such that

$$\int_0^a x\ \mathrm{d}x = \int_0^a \sqrt{x}\ \mathrm{d}x.$$

(2)

(*c*) Find the exact value of a.

(4)

(*d*) Hence, or otherwise, find a non-constant function $\mathrm{f}(x)$ and a constant b ($b \neq 0$) such that

$$\int_{-b}^{b} \mathrm{f}(x)\ \mathrm{d}x = \int_{-b}^{b} \sqrt{[\mathrm{f}(x)]}\ \mathrm{d}x.$$

(2)

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3. (*a*) Solve, for $0 \leq x < 2\pi$,

$$\cos x + \cos 2x = 0.$$

(5)

(*b*) Find the exact value of x, $x \geq 0$, for which

$$\arccos x + \arccos 2x = \frac{\pi}{2}.$$

(6)

[$\arccos x$ is an alternative notation for $\cos^{-1} x$.]

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4. The function $\mathrm{h}(x)$ has domain $\mathbb{R}$ and range $\mathrm{h}(x) > 0$, and satisfies

$$\sqrt{\int \mathrm{h}(x)\ \mathrm{d}x} = \int \sqrt{\mathrm{h}(x)}\ \mathrm{d}x.$$

(*a*) By substituting $\mathrm{h}(x) = \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2(y + c),$$

where c is constant.

(5)

(*b*) Hence find a general expression for y in terms of x.

(4)

(*c*) Given that $\mathrm{h}(0) = 1$, find $\mathrm{h}(x)$.

(2)

5.

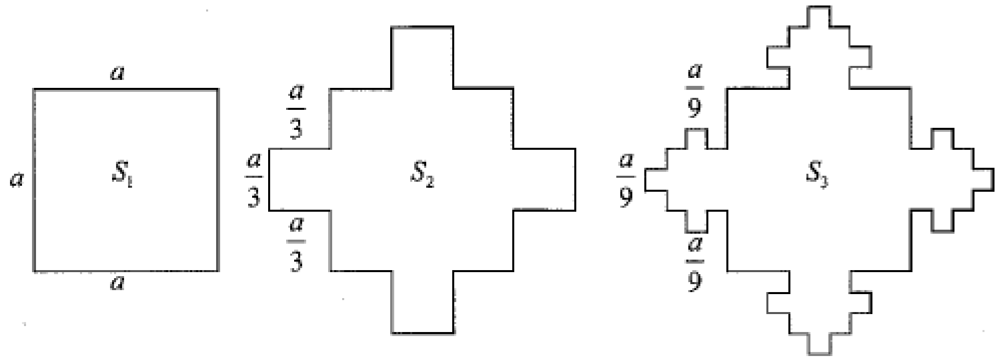

Figure 1

Figure 1 shows part of a sequence $S_1, S_2, S_3, \ldots,$ of model snowflakes. The first term S_1 consists of a single square of side a. To obtain S_2, the middle third of each edge is replaced with a new square, of side $\frac{a}{3}$, as shown in Figure 1. Subsequent terms are obtained by replacing the middle third of each external edge of a new square formed in the previous snowflake, by a square $\frac{1}{3}$ of the size, as illustrated by S_3 in Figure 1.

(*a*) Deduce that to form S_4, 36 new squares of side $\frac{a}{27}$ must be added to S_3.

(1)

(*b*) Show that the perimeters of S_2 and S_3 are $\frac{20a}{3}$ and $\frac{28a}{3}$ respectively.

(2)

(*c*) Find the perimeter of S_n.

(4)

(*d*) Describe what happens to the perimeter of S_n as n increases.

(1)

(*e*) Find the areas of S_1, S_2 and S_3.

(2)

(*f*) Find the smallest value of the constant S such that the area of $S_n < S$, for all values of n.

(5)

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6. **Figure 2**

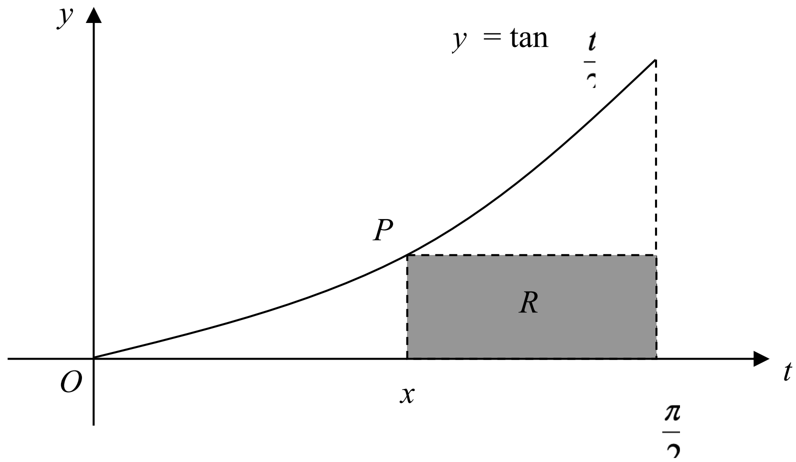

Figure 2 shows a sketch of the curve C with equation $y = \tan \frac{t}{2}$, $0 \leq t \leq \frac{\pi}{2}$.

The point P on C has coordinates $\left(x, \tan \frac{x}{2}\right)$.

The vertices of rectangle R are at $(x, 0)$, $\left(\frac{x}{2}, 0\right)$, $\left(\frac{x}{2}, \tan \frac{x}{2}\right)$ and $\left(x, \tan \frac{x}{2}\right)$ as shown in Figure 2.

(*a*) Find an expression, in terms of x, for the area A of R.

(1)

(*b*) Show that $\frac{dA}{dx} = \frac{1}{4}(\pi - 2x - 2\sin x)\sec^2 \frac{x}{2}$.

(4)

(*c*) Prove that the maximum value of A occurs when $\frac{\pi}{4} < x < \frac{\pi}{3}$.

(7)

(*d*) Prove that $\tan \frac{\pi}{8} = \sqrt{2} - 1$.

(3)

(*e*) Show that the maximum value of $A > \frac{\pi}{4}(\sqrt{2} - 1)$.

(2)

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7. The points O, P and Q lie on a circle C with diameter OQ. The position vectors of P and Q, relative to O, are **p** and **q** respectively.

(a) Prove that $\mathbf{p.q} = |\mathbf{p}|^2$.

(3)

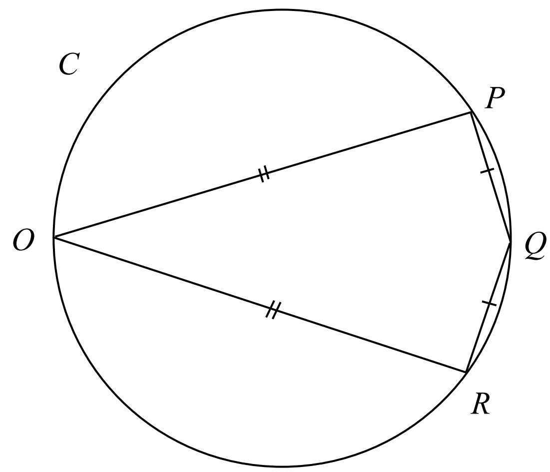

Figure 3

The point R also lies on C and $OPQR$ is a kite K as shown in Figure 3. The point S has position vector, relative to O, of $\lambda\mathbf{q}$, where λ is a constant. Given that $\mathbf{p} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{q} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and that OQ is perpendicular to PS, find

(*b*) the value of λ,

(2)

(*c*) the position vector of R,

(3)

(*d*) the area of K.

(4)

Another circle C_1 is drawn inside K so that the 4 sides of the kite are each tangents to C_1.

(*e*) Find the radius of C_1 giving your answer in the form $(\sqrt{2} - 1)\sqrt{n}$, where n is an integer.

(5)

A second kite K_1 is similar to K and is drawn inside C_1.

(*f*) Find that area of K_1.

(3)

MARKS FOR STYLE, CLARITY AND PRESENTATION: 7 MARKS
TOTAL FOR PAPER: 75 MARKS

END